Dielectric waveguide model for guided surface polaritons

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Although surface polariton modes supported by finite-width interfaces can guide electromagnetic energy in three dimensions, we demonstrate for the first time to our knowledge that such modes can be modeled by the solutions of two-dimensional dielectric slab waveguides. An approximate model is derived by a ray-optics interpretation that is consistent with previous investigations of the Fresnel relations for surface polariton reflection. This model is compared with modal solutions for metal stripe waveguides obtained by full vectorial magnetic-field finite-difference methods. The field-symmetric modes of such waveguides are shown to be in agreement with the normalized dispersion relationship for analogous TE modes of dielectric slab waveguides. Lateral confinement is investigated by comparison of power-density profiles, and implications for the diffraction limit of guided polariton modes are discussed. © 2005 Optical Society of America

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Surface plasmon polaritons and surface phonon polaritons have received much attention for their ability to guide electromagnetic energy.[1–3] Unlike dielectric waveguides, which confine volume electromagnetic waves to an optically dense core, these surface electromagnetic waves are localized at interfaces between dielectric materials and metals or ionic solids that support charge density oscillations. This surface localization has led researchers to explore the potential for transporting information via guided polariton modes with smaller spatial extents than can be achieved with diffraction-limited dielectric waveguides.[4]

The best-studied guided polariton modes involve surface plasmon polaritons supported by finite-width metal stripes.[5] Although such modes have proved difficult to calculate, recent characterization by near-field scanning optical microscopy has probed their localized light intensities.[6] Based on published bound modal solutions, initial interpretation of these images led to conclusions that surface plasmon modes are inconsistent with dielectric waveguide theory. However, recent solutions for the experimentally relevant leaky modes may provide for an interpretation that is consistent with ray optics.[7] In this Letter we investigate the applicability of dielectric waveguide theory to surface polariton modes along finite-width interfaces. By combining previous research on the optics of surface polaritons[8] with a ray-optics model for guided waves, we demonstrate that an equivalent dielectric slab waveguide can be used to approximate the solutions of guided polariton modes.

By definition, a guided mode is an eigenstate of the electromagnetic field that propagates in a specified direction (e.g., z) with a unique propagation constant (i.e., $k_z \equiv \beta + i\alpha$). For dielectric slab waveguides an exact analytical formulation for the modal solutions is possible. Nevertheless, to acquire a physical intuition for these waveguides, a ray-optics interpretation is often used. In this model the guided mode is defined by the superposition of plane-wave solutions, which constructively interfere in the high-index region after total internal reflection (TIR). These plane-wave solutions have a wave-vector magnitude determined by the core's refractive index [i.e., $|k| = (\omega/c)\sqrt{\epsilon_{\text{core}}} = k_0 n_{\text{core}}$]. Thus the calculation of the phase and the amplitude of reflections at each interface can be obtained from the Fresnel relations.

For surface polariton reflection at the edge of two dielectric regions, it has been shown that Fresnel-like relations provide good approximations when the in-plane wave vectors for infinitely wide polariton modes (k_{sp}) are considered.[9] For example, polariton TIR occurs when the projection of the incident wave vector along such an edge exceeds the maximum magnitude allowed for a transmitted polariton. Conservation of momentum therefore stipulates that an associated critical angle [$\theta_c = \sin^{-1}(k_{\text{sp},t}/k_{\text{sp},i.})$] can be anticipated. This analysis is equivalent to treatment of each surface polariton region with an effective refractive index, defined as

$$n_{\text{eff}} = k_{\text{sp}}/k_0, \tag{1}$$

where k_{sp} is the in-plane wave vector of a surface polariton supported by an infinitely wide structure.

Here we extend the above analysis to consider surface polariton TIR at the metal film edge depicted in Fig. 1(a). The effective refractive index of the polariton (n_{eff}) represents the magnitude of the incident in-plane wave vector, whereas the refractive index of the dielectric region ($n_d = \sqrt{\epsilon_d}$) represents the wave vector of a transmitted homogeneous wave. Thus a critical angle [$\theta_c = \sin^{-1}(n_d/n_{\text{eff}})$] is expected. If such an effective-index treatment accurately represents the internally reflected surface polariton, an approximate ray-optics model for guided polariton modes can be derived. For the continuous and constructive TIR of polaritons along a finite-width interface [shown in Figs. 1(b) and 1(c)], the guided mode resembles, and is modeled by, the transverse electric (TE) mode of an equivalent dielectric slab waveguide [shown in Fig. 1(d)]. We call this model approximate because the transverse magnetic nature of a surface polariton requires electric-field components both normal to the supporting surface and along the direction of propagation. However, for surface polaritons that propa-

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gate any significant distance, the dominant electric-field component is normal to the film as drawn in Fig. 1(c), and thus the analogous dielectric waveguide modes in our two-dimensional model should be TE.

To investigate this model we compare its approximate solutions with those recently published by Al-Bader.[10] These solutions were obtained by a full-vectorial magnetic-field finite-difference method (FVH-FDM), which combines piecewise application of the Helmholtz equation with explicit considerations for the boundary conditions of Maxwell's equations.[11] We consider as a function of varying thickness (t), the surface plasmon modes supported by coupled top and bottom interfaces of a metal stripe embedded within a dielectric matrix (as shown in the inset of Fig. 2). The width of the model waveguide is identical to the finite width (W) of the stripe, and the effective refractive index of the core is determined by Eq. (1). However, it is important to note that the wave vector along an infinitely wide structure (k_{sp}) depends on the spatial separation (t) of the coupled interfaces. So, one must determine the value of k_{sp} for Eq. (1) by solving for the modes of a two-dimensional metallic slab waveguide.[12] Here the solutions for both the field-symmetric and -antisymmetric modes were determined by use of the reflection pole method.[13]

In Fig. 2 the solutions for the associated TE modes of our model are plotted relative to the FVH-FDM solutions published in Ref. 10. Note that the two lowest-order stripe modes with field-symmetric profiles in the vertical dimension (M_{00} and M_{10} in the notation of Al-Bader) are well approximated by the equivalent TE_0 and TE_1 modes of a slab waveguide. The fact that this agreement with the real (β) and the imaginary (α) components of the propagation constant occurs for most stripe thicknesses demonstrates the model's applicability for a variety of effective refractive indices. In this Letter we limit our analysis to these field-symmetric modes. Note that our modeled solutions also predict the expected decoupling of surface plasmons along the top and bottom interfaces for thick waveguides but, in so doing,

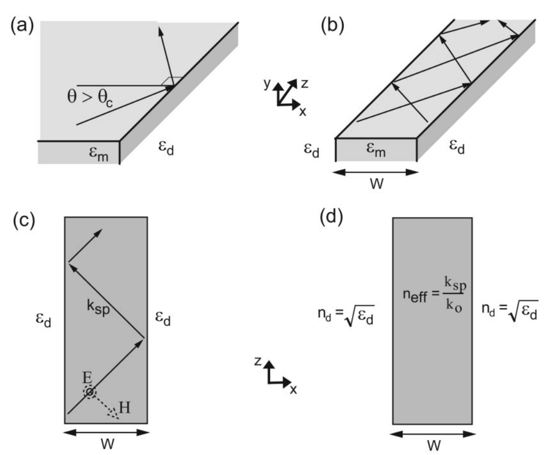

Fig. 1. Dielectric waveguide treatment of surface polaritons along finite interfaces: (a) TIR of a surface polariton wave, (b) ray-optics interpretation of a surface polariton mode, (c) top view of the ray-optics interpretation, (d) equivalent two-dimensional dielectric slab waveguide.

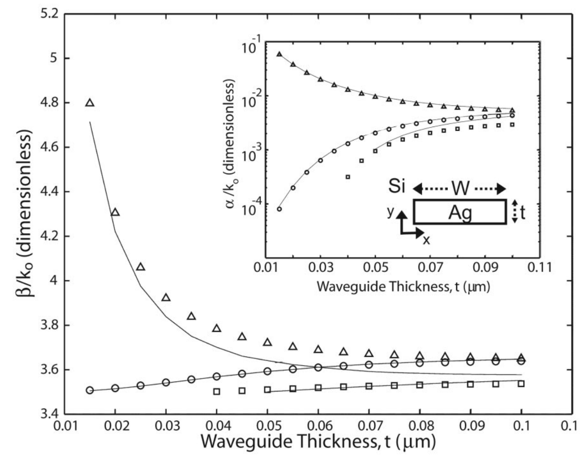

Fig. 2. Surface plasmons supported by coupled interfaces of a finite-width silver stripe of varying thickness (t) embedded in a silicon matrix. Markers denote approximate solutions obtained from the dielectric waveguide model for M_{00} (circles), M_{01} (triangles), and M_{10} (squares) modes. Solid curves denote solutions previously presented by J. Al-Bader.[10] Relevant parameters are $W=1\ \mu\mathrm{m}$, $\lambda=1.55\ \mu\mathrm{m}$, $\epsilon_{\mathrm{Ag}}=-125.735+i3.233$, $\epsilon_{\mathrm{Si}}=12.25$.

disagree with the field-antisymmetric M_{01} mode described in Ref. 10 that does not converge with the M_{00} mode for large t. This discrepancy may result from discretization error introduced by modeling the rapidly varying fields of field-antisymmetric modes with a finite-difference scheme in the y direction.[14]

To extend our investigation we implemented a FVH-FDM to solve for the M_{00} and M_{10} modes as a function of variation in stripe width and thickness. To facilitate comparison with the dielectric waveguide model we present these solutions in the normalized form for which universal plots of the dispersion relations of TE slab modes exist.[15] The normalized frequency (V) and the normalized guide index (b) are defined as

$$V \equiv k_0 W ({n_{eff}}^2 - {n_d}^2)^{1/2}, \tag{2}$$

$$b \equiv \frac{[(\beta + i\alpha)^2/k_0^2] - {n_d}^2}{{n_{eff}}^2 - {n_d}^2}, \tag{3}$$

where $(\beta+i\alpha)$ is the propagation constant of the three-dimensional guided polariton mode and n_{eff} is the effective refractive index for our two-dimensional dielectric model as defined by Eq. (1).

Figure 3 shows that the lowest-order field-symmetric modes of a metal stripe waveguide are in good agreement with the universal solutions for the equivalent TE dielectric slab waveguide. This agreement extends to the prediction of the cutoff frequency for the higher-order (M_{10}) mode. The added value of this normalized representation is a basis on which to evaluate the applicability of our dielectric waveguide model. The maximum deviation from this model occurs for small values of the normalized guide index, for which small deviations in normalized frequency result in large changes to the guide index. Although the approximation appears to be most accurate when the optical mode is confined laterally to the effective core region of the dielectric model (i.e., $b \rightarrow 1$), it should also be noted that the normalized guide index (b) scales with the effective refractive index (n_{eff}). Hence, even as the optical fields become less confined to the supporting interfaces ($n_{\mathrm{eff}} \rightarrow n_d$), this model continues to approximate accurately ever-smaller propagation constants ($\beta + i\alpha$).

Moreover, the lateral confinement of the three-dimensional guided polariton is well predicted by the two-dimensional model. As a representative example we have plotted in the inset of Fig. 3 the lateral power density [i.e., $\int \Re(S_z) \mathrm{d}y$] for a metal stripe waveguide and for the equivalent dielectric waveguide. Despite the discontinuity of the Poynting vector for the polariton mode, the dielectric approximation anticipates the power-density profile. That such physical behavior for a three-dimensional surface wave can be predicted by a volume electromagnetic waveguide has implications for many debated topics in guided polariton optics. For example, the minimum optical mode size of a polariton waveguide has been a subject of much interest and, like that of a dielectric waveguide, must be determined by an uncertainty principle. Without validation of an appropriate wave-vector basis set, it is difficult to formulate such a diffraction limit.[12] However, based on index guiding, a diffraction-limited mode size (Δx) in the lateral dimension can be derived for modes accurately approximated by our model as

$$\Delta x \geqslant \lambda_0 / 2 n_{\mathrm{eff}}. \tag{4}$$

In conclusion, we have presented a physically intuitive dielectric waveguide model for surface polariton modes supported by finite-width interfaces. This model was formulated by consideration of earlier research on the optics of surface polaritons, and its demonstrated applicability coincides with cases for which surface polariton reflection is well approximated by Fresnel-like relations. We anticipate that in the future this model could be refined by study of the vertical polariton modes excited by such reflections.[16] Although its description has been omitted owing to limited space, this model has also been used to predict the leaky and bound modes supported by single interfaces of finite width. As presented, though, this Letter should facilitate a view of guided polariton optics that is consistent with conventional guided-wave phenomena, and the physical model provided offers a basis from which to leverage the decades of research on dielectric integrated optics for the development of analogous plasmonic devices.

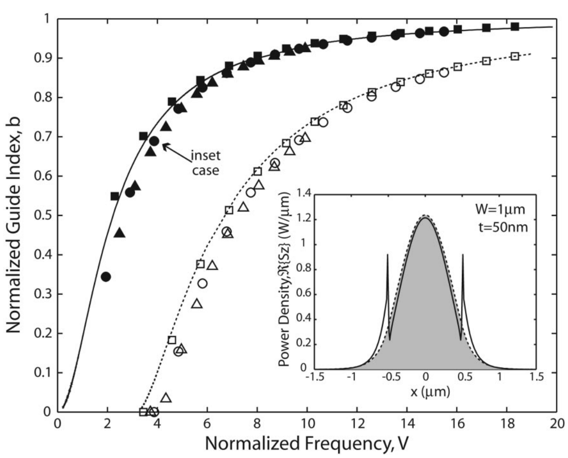

Fig. 3. Normalized dispersion curves for surface plasmon modes of silver stripe waveguide (as in Fig. 2). Filled and open markers denote solutions obtained by FVH-FDM for the M_{00} and M_{10} modes, respectively. For three stripe thicknesses [$t = 25$ nm (triangles), 50 nm (circles), and 100 nm (squares)], width W varied from 0.5 to 4 μm. Solid and dashed curves represent the TE_0 and TE_1 modes of a dielectric slab waveguide, respectively.[15] Inset, comparison of the lateral power densities for a surface plasmon waveguide (solid curve, $W = 1$ μm; $t = 50$ nm) and an approximate dielectric waveguide (dashed curve with gray shading, $n_{\mathrm{eff}} = 3.628 + i0.002267$) normalized to unit power.

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